[07-09-09D-T10]

Absolute value inequalities involving quadratic expressions

■ **[EX1] Find all values of x for which $|x^2 + 5x + 10| > 4$ is true.**

Solution.

Eliminating the absolute value bars results in two cases. They are:

$x^2 + 5x + 10 > 4$ or $x^2 + 5x + 10 < -4$

Case 1.

$x^2 + 5x + 10 > 4$

$\Longleftrightarrow x^2 + 5x + 6 > 0$

$\Longleftrightarrow (x + 3)(x + 2) > 0$

$\Longleftrightarrow (x + 3 > 0 \text{ and } x + 2 > 0)$ OR $(x + 3 < 0$ and $x + 2 < 0)$

Hence,

Case [1.1] $x + 3 > 0$ and $x + 2 > 0$

$x + 3 > 0 \Longrightarrow x > -3$ and $x + 2 > 0 \Longrightarrow x > -2$

$x \in (-3, \infty +) \bigcap (-2, \infty +) = (-2, \infty +)$

OR

Case [1.2] $x + 3 < 0$ and $x + 2 < 0$

$x + 3 < 0 \Longrightarrow x < -3$ and $x + 2 < 0 \Longrightarrow x < -2$

$x \in (-\infty, -3) \bigcap (-\infty, -2)$
$x \in (-\infty, -3)$

So, in Case 1

$x \in (-\infty, -3) \bigcup (-2, \infty +)$

Case 2.

$x^2 + 5x + 10 < -4$

$\Longleftrightarrow x^2 + 5x + 14 < 0$

Since $b^2 - 4(ac) = 25 - 4(14) < 0$, there is no solution in this case.

So, in Case 2

$x \in \emptyset$

Since Case $1 \Longrightarrow x \in (-\infty, -3) \bigcup (-2, \infty +)$ and Case $2 \Longrightarrow x \in \emptyset$ and either Case 1 or Case 2 is true,

$x \in (-\infty, -3) \bigcup (-2, \infty +) \bigcup \emptyset$

$\therefore$ x∈(-∞,-3)⋃(-2,∞+) .